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Alternative fuel station location model with demand learning

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Abstract In this paper, we study the optimal location decision for a network of alternative fuel stations (AFS) servicing a new market where the demand rate for the refueling service can be learned over time. In the presence of demand learning, the firm needs to make a decision, whether to actively learn the market through a greater initial investment in the AFS network or defer the commitment since an overly-aggressive investment often results in sub-optimal AFS locations. To illustrate this trade-off, we introduce a two-stage location model, in which the service provider enters the market by deploying a set of stations in the first stage under uncertainty, and has the option to add more stations in the second stage after it learns the demand. The demand learning time (length of the first stage) is endogenously determined by the service provider's action in the first stage. To solve this problem, we develop an efficient solution method that provides a framework to achieve a desired error rate of accuracy in the optimal solution. Using numerical experiment, we study the trade-off between active learning and deferred commitment in AFS deployment strategy under different market characteristics. Further, we find that the lack of planning foresight typically results in an over-commitment in facility investment while the service provider earns a lower expected profit.

Keywords Alternative fuel station operations · Facility location · Maximal covering problem · Demand learning

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1 Introduction

Alternative fuel vehicles (AFVs) are vehicles that run on fuels derived from sources other than petroleum (petrol or diesel); examples include electric, hydrogen, and bio-diesel vehicles. With growing concerns over energy security (resulting from rapidly increasing/fluctuating oil prices) and environmental issues (such as CO_2 emissions and air pollution), AFVs have become a high priority for many governments and car manufacturers around the world in search for cleaner transportation solutions. However, adoption of AFVs often suffers from the lack of supporting infrastructure, in particular, alternative fuel stations (AFS) at which AFV can be refueled. To support mass adoption of AFVs, the AFS infrastructure must be deployed optimally and economically, to ensure extensive geographical coverage and convenience to drivers, while keeping operating costs low enough to be competitive with existing gasoline counterparts. In this paper, we consider the problem of optimally deploying a network of AFS to service a growing AFV market.

In deploying the AFS infrastructure, facility location decision (number and locations of the stations) is one of the most important factors because the locational convenience (i.e., distance) to the potential AFV buyers plays a big role in the adoption process. While the location decision represents great opportunity for the refueling service providers, it also implies great risk since it involves a significant commitment of resources for a long period of time. When the firm decides to open a new station, it has to make a substantial initial investment; either buy and develop the real estate or commit to a long-term lease typically ranging from 5 to 20 years (Levy and Weitz 2008). Thus, unlike poor pricing or operational decisions, poor location decisions negatively affect the firm for an extended period of time.

The risk of commitment in capital-intensive AFS business increases even further when it comes to entering a new market with high uncertainties. The firm may face various sources of uncertainties resulting from local economy and/or consumer behaviors that affects the AFV adoption as well as the resulting demand for AFS service. Although the service provider can conduct market research (such as pilot testing or market surveys) to reduce such uncertainties, it is difficult to fully understand the usage behavior of potential AFV drivers without actually operating the AFS. Hence, the service providers deploy AFS in a *dynamic* fashion as they *learn* the demand over time. Interestingly, in practice, we observe that different firms adopt different market entry approaches and, accordingly, the resulting AFS deployment decisions vary significantly. For example, Tesla chose to slowly expand its network of AFS (i.e., superchargers for its electric vehicles), because of the high uncertainty in the mass-market demand, only operating 8 stations across the nation (CNN 2013). Its strategy is to plan for expansion after learning about the purchasing and usage patterns more closely after the initial introduction stage (MIT Technology Review 2013). In contrast, before fully learning the demand, Better Place decided to make a significant initial capital commitment to roll out nation-wide battery switching station networks in full scale for its electric vehicle drivers in Israel and Denmark (Jerusalem Post 2011).

In this paper, we consider the AFS location problem where the uncertain market characteristic can be learned over time. In particular, we study the service provider's optimal station location decision in which the time to learn the AFV refuel service demand rate depends on the firm's initial station location decision. With the option to defer part of investments until fully learning the service demand, the firm needs to make an AFS deployment decision. That is, should the firm actively learn the market through a greater initial investment (e.g., open more stations), since service demand data can be collected at a faster rate? Or should the firm defer the commitment since an overly-aggressive investment often results in suboptimal locations, which will adversely affect its performance in the long run. The primary objective of this study is to understand optimal location decision in the presence of such demand learning trade-off.

To this end, we introduce a two-stage AFS location model that captures the *demand learning effect*. We consider the demand rate in the service region to be uncertain, but can be learned during the first stage. After the first stage, we assume that the demand rate for the AFV refueling service is fully learned and that the service provider has the option to locate additional stations in the second stage. We let the duration of the first stage (demand learning time) to be *endogenously* determined as a function of service provider's action in the first stage. Therefore, the service provider can choose to either shorten the demand learning time by aggressively investing upfront or defer the commitment at the cost of delaying the demand learning time.

The main contribution of this research is two-fold. First, we develop an efficient and effective solution method for solving the two-stage AFS location problem with demand learning (Sect. 4.2). The proposed solution method offers a service provider's approximated expected profit within the error rate of ϵ provided by the network designer. Also, this solution method is applicable to any location problem with an endogenous learning time. We test the algorithm on a network constructed using census data from the City of Chicago (Sect. 4.2). Second, we derive relevant managerial insights regarding AFS location decision. We show that the firm's optimal AFS deployment strategy depends on the speed of the learning in the local service region (Sect. 5.1). Further, we show that lack of planning foresight typically results in over-commitment in AFS infrastructure investment while the firm earns a lower expected profit (Sect. 5.2).

The remainder of this paper is structured as follows. In Sect. 2, we review the related literature. In Sect. 3, we introduce the two-stage AFS location problem with demand learning. We also specify the interesting structural properties of the problem. In Sect. 4, we develop a solution method for the proposed problem and report the computation results. In Sect. 5, we first study the impact of demand learning by obtaining a firm's optimal strategy under different market characteristics. Then, we study the value of foresight in location planning by contrasting the optimal decision to a myopic decision. Finally, we conclude the paper by summarizing the managerial insights and proposing directions for future research in Sect. 6.

2 Literature review

To capture the degree of refuel service demand of AFV, we employ facility location model (see, e.g., Hale and Moberg 2003 for a review), and the coverage model in particular. Church and ReVelle (1974) introduce the Maximal Coverage Location Problem [MCLP] which finds the locations of a given number of facilities to maximize the total number of demands served by the set of opened facilities. This problem assumes a binary coverage scheme; i.e., the service is determined to be adequate if the demand is within a given distance and is considered inadequate if the distance exceeds some critical value. Daskin (1983) extends the problem to the "expected" covering case by taking into account the possibility of facility congestion. Drezner (2009) considers the problem of locating retail stores in an uncertain market environment to cover as much demand as possible.

As a refinement of the covering location problem, Berman and Krass (2002) consider a generalized version of the MCLP which allows for partial coverage of consumers instead of binary coverage. Berman et al. (2003) and Drezner et al. (2004) discuss the Gradual Covering Location Problem [GCLP] in which the extent of demand coverage is defined as a function of the consumer's traveling distance. In particular, they consider the lower and upper thresholds in traveling distance; consumers who have a traveling distance less than the lower threshold are fully covered whereas consumers who live farther than the upper threshold are not covered. The coverage for consumers located in between gradually decreases as a function of the traveling distance. Drezner et al. (2010) extend the GCLP to the stochastic case when the upper and lower distances are random variables. We consider a similar coverage scheme in this paper. We assume that the coverage function is a non-increasing function of the distance between a demand node and its closest facility. For more details on the coverage location problems, please see Jacobson (1990) (for discrete models) and Plastria (2002) (for continuous models).

On locating the AFS network system (to maximize the coverage of its refueling service demand), however, there has only been limited number of studies. Kuby and Lim (2005) study a flow-refueling location problem for AFV. The main focus is to capture as much AFV traveling paths as possible (so that vehicles can be refueled before running out of fuel), by locating a given number of refuel stations. In Kuby and Lim (2007), the earlier work is extended by allowing location of stations along arcs, in addition to nodes, in the network. Upchurch et al. (2009) consider the problem incorporating limited capacity for the refuel stations. In this paper, we consider a demand coverage model for AFS deployment problem where the service provider wants to maximize the total coverage for its service users (i.e., AFV owners). Similar to the coverage model proposed by Drezner et al. (2004), we assume that the coverage function (on the amount of AFS service demand) is a non-increasing function of the distance between a demand node and its closest facility.

Facilities typically function for an extended period of time, during which a certain aspect of market environment may be learned (Snyder 2006). For this reason, many facility location problems involve an extended planning horizon where firms make a set of dynamic decisions over time. Ballou (1968), Wesolowsky (1973), and Daskin et al. (1992) provide pioneering work on the dynamic facility location problem. Dynamic location problems provide a set of plans that involve expanding facilities and/or relocating existing facilities as uncertain information such as demand, travel cost, and competition unveil over time. Van Roy and Erlenkotter (1982) and Baron et al. (2010) consider a facility location problem on a dynamic setting where demand changes over time. Campbell (1990) studies the dynamic location of transportation terminals where demand, transportation cost and the facility cost alter over time. In this paper, we assume the refueling service demand rate to be the uncertain factor and that a monopoly firm dynamically deploys its AFS to maximize its expected profit.

We limit the problem to a two-stage setting (with an infinite time horizon) since it suffices to the value of demand learning in whether the service provider should actively deploy the facility (in the first stage) or defer the commitment until the demand is learned (and deploy in the second stage). On studying the dynamics of facility location problem in two stages, Current et al. (1997) consider two versions of problems where the total number of facilities to open varies depending on the future scenario. Berman and Drezner (2008) also study a two-stage problem with a fixed number of facilities opened in the first stage. They attempt to minimize the total cost of serving all the demand, while keeping in view that additional facilities can be opened in the future stage. More recently, Shu (2010) study an integrated location and two-echelon inventory model where location decisions are made in the first stage and inventory replenishment and allocation decisions are made in the second stage to minimize the expected system cost. Wang et al. (2013) propose a continuum approximation framework to solve a dynamic facility location problem based on the projection of future demand. Although these papers share the similar feature of dynamic location model (mostly on a two-stage setting) with demand uncertainty, they do not capture the learning effect. In particular, the length of each stage is exogenously given and the firm passively makes decision under a given setting. In this paper, we explicitly incorporate the demand learning effect by assuming the learning time (length of the first stage) is endogenously determined as a function of firm's first-stage actions, the amount of first-stage demand coverage in particular. In the sense that the firm has to make such demand coverage decision, our model also has some similarity to the model from Shen (2006) in which the firm is determining which and how many customers to serve designing supply chain network.

Hiller and Shapiro (1986) and Rob (1991) are the first ones to consider *learning* in a firm's capacity expansion. The optimal timing of operational decisions has also been studied in the various fields of OR/OM in recent years; for example, Ke et al. (2013) study the optimal timing to release a second version of a product considering product diffusion and inventory costs. However, the learning issue in the facility location setting has not received much attention. Our work attempts to partially fill this gap by studying the optimal timing decisions in an emerging context of AFS location, taking into account the important, yet under-studied aspect of demand learning.

3 Dynamic AFS location model

3.1 Single-stage model

To illustrate the problem setting, we first present a single-stage AFS location problem without demand learning. Consider a service provider who enters a service region which is characterized by a network $G(V, A)$ where V is the set of nodes and A is the set of arcs. We consider $V = \{I \cup J\}$ where $I = \{1, \dots, m\}$ is the set of attraction nodes and $J = \{1, \dots, n\}$ is the set of sites where the AFS can be located. We do not require the two sets, I and J, to be disjoint. In each attraction node i, we assume h_i to be the frequency of visits by all vehicle types. To capture the uncertainty in AFS service demand, we introduce a random variable θ and refer to this as *service demand rate*. We assume the service demand rate is identical across the network of the service region; i.e., θ is independent of i. Hence, θh_i is the potential AFV refueling service sales per unit time for each attraction node i.

As the AFS may not be located at the attraction node i, AFV drivers may need to take detours to access the AFS. Naturally, the actual demand from node i captured by an AFS at location j is decreasing in the detour distance, i.e., the distance between i and j. Therefore, we consider the demand at node $i \in I$ to be only partially covered in a proportion given by the coverage function $g_i(d) \in [0, 1]$ where d is the distance to its closest opened facility. Similar to Berman and Krass (2002) (as well as Berman et al. (2003), Drezner et al. (2004), and Drezner et al. (2010)), we assume that the demand coverage function $g_i(d)$ is a non-increasing convex function of d with $g_i(0) = 1$ for all $i \in I$. Hence, the actual sales per unit time at node i can be expressed as $\theta h_i g_i(d_i(X))$ where X is the set of opened AFS and $d_i(X) = \min_{j \in X} d(i, j)$. Denoting the revenue per unit service sales by r, total revenue per unit time is $\sum_{i \in I} r\theta h_i g_i(d_i(X))$. We denote the fixed cost for operating the set of AFS X per unit time by $f(X)$, where $f(.)$ is a modular function. Revenue and cost are discounted over an infinite time horizon with a discount rate denoted by α. Then, the Single-stage AFS Location Problem [SALP] with a given service demand rate that maximizes the firm's profit can be formulated as follows:

$$\textbf{[SALP]} \quad \max_{X\subset J}\left\{\int_0^{\infty} e^{-\alpha t}\left(\sum_{i\in I} r\theta h_i g_i\big(d_i(X)\big) - f(X)\right)dt\right\}$$
$$= \max_{X\subset J}\left\{\frac{1}{\alpha}\left(\sum_{i\in I} r\theta h_i g_i\big(d_i(X)\big) - f(X)\right)\right\}. \tag{1}$$

We note that the single-stage AFS location problem can be transformed to the Uncapacitated Fixed-charge Location Problem [UFLP].

Remark 1 The Single-stage AFS Location Problem [SALP] is reducible to the Uncapacitated Fixed-charge Location Problem [UFLP].

Proof Let H represent the maximum total service demand rate of the service region per unit time, $H = \sum_{i\in I} h_i$. Define a new distance metric as $\widetilde{d}_i(X) = \max_{j\in X}[1 - g_i(d(i,j))]$. Then, it follows that $\widetilde{d}_i(X) = \max_{j\in X}[1 - g_i(d(i,j))] = 1 - \min_{j\in X} g_i(d(i,j)) = 1 - g_i(\min_{j\in X} d(i,j)) = 1 - g_i(d_i(X))$. Thus, for any $X \subset J$,

$$\begin{aligned}\frac{1}{\alpha}\left(\sum_{i\in I} r\theta h_i g_i\big(d_i(X)\big) - f(X)\right) &= \frac{1}{\alpha}\left(\sum_{i\in I} r\theta h_i\big(1 - \big(1 - g_i\big(d_i(X)\big)\big)\big) - f(X)\right)\\ &= \frac{1}{\alpha}\left(r\theta H - r\theta\sum_{i\in I} h_i\big(1 - g_i\big(d_i(X)\big)\big) - f(X)\right)\\ &= \frac{1}{\alpha}\left(r\theta H - r\theta\sum_{i\in I} h_i\widetilde{d}_i(X) - f(X)\right).\end{aligned}$$

Since H, θ and r are constants, maximizing this problem is equivalent to the following UFLP:

$$\min_{X\subset J}\left\{f(X) + k\sum_{i\in I} h_i\widetilde{d}_i(X)\right\}$$

where k is a constant. □

This is an interesting and useful result since UFLP, while NP-Hard, has many practical solution methods available (Daskin 1995). This relationship is also in line with the connection between coverage problems and median problems (of which the UFLP is an extension) identified previously in the literature (Church and Weaver 1986).

Now, we consider the case in which the service demand rate θ is unknown. We assume θ follows a certain distribution with mean of $\bar{\theta}$ and standard deviation of σ. Given the distribution, one can then consider a problem of maximizing the *expected* profit as follows:

$$\textbf{[SALP-U]} \quad \max_{X\subset J}\left\{\mathbb{E}_{\theta}\left[\frac{1}{\alpha}\left(\sum_{i\in I} r\theta h_i g_i\big(d_i(X)\big) - f(X)\right)\right]\right\}. \tag{2}$$

Using this problem as a building block, we introduce a two-stage AFS location problem that incorporates *service demand learning* in the next subsection.

3.2 Two-stage model with learning

In the two-stage model, the subject of learning is the demand rate for the AFS service for the given region which, in turn, determines the actual service sales for each node. In the first stage, the demand rate, θ, is uncertain, but its distribution is known (as in [SALP-U] (2)). Based on its distribution, the service provider must decide in advance where and how many stations to open, X^1, taking into account the next stage. In Stage 2, we assume the service demand rate is fully learned, i.e., the precise value of θ is realized. Upon the realization of θ and given X^1, the service provider deploys additional stations accordingly, X^2, (as in the [SALP] (1)) to maximize its total profit. The two stages reflect the firm's strategic location decisions at the early and mature stages of entering a market, respectively. To focus on these strategic nature of location decisions, we do not consider the possibility of closing or relocating the stations opened at the previous stage (e.g., Farahani et al. 2009). In the literature, closure and relocation of facilities are typically assumed to be impossible because doing so often incurs high cost (Current et al. 1997; Baron et al. 2010), which is especially true for the case of AFS. Further, to focus on the learning effect in facility deployment strategy, we do not consider the possibilities of the service provider conducting other means of market research to learn θ. Denoting the demand learning time by T, the Two-stage AFS Location Problem for a given T [TALP] is formulated as follows:

$$\max_{X^1 \subset J} \left\{ \mathbb{E}_\theta \left[\int_0^T e^{-\alpha t} \left(\sum_{i \in I} r\theta h_i g_i \left(d_i\left(X^1\right)\right) - f\left(X^1\right) \right) dt + V\left(X^1, \theta\right) \right] \right\} \tag{3}$$

where $V(X^1, \theta)$ is the optimal objective value of:

$$\max_{X^2 \subset J \setminus X^1} \left\{ \int_T^\infty e^{-\alpha t} \left(\sum_{i \in I} r\theta h_i g_i \left(d_i\left(X^1 \cup X^2\right)\right) - f\left(X^1 \cup X^2\right) \right) dt \right\}. \tag{4}$$

Let $X^2(X^1, \theta)$ be the optimal solution to (4), which is a function of X^1 and the realization of θ. To solve [TALP], $X^2(X^1, \theta)$, as a policy, has to be jointly optimized with X^1. Therefore, with some algebraic work, the problem can be reexpressed as follows:

$$\textbf{[TALP]} \quad \max_{\substack{X^1 \subset J \\ X^2(X^1,\theta) \subset J \setminus X^1, \forall X^1, \theta}} \frac{1}{\alpha} \left\{ \mathbb{E}_\theta \left[\left(1 - e^{-\alpha T}\right) \sum_{i \in I} r\theta h_i g_i \left(d_i\left(X^1\right)\right) - f\left(X^1\right) \right.\right.$$
$$\left.\left. + e^{-\alpha T} \left(\sum_{i \in I} r\theta h_i g_i \left(d_i\left(X^1 \cup X^2\left(X^1, \theta\right)\right)\right) - f\left(X^2\left(X^1, \theta\right)\right) \right) \right] \right\}. \tag{5}$$

By extending the problem to a two-stage setting, we note that the service provider now may have incentive to deploy fewer AFS in the first stage because it has an option to deploy more in the second stage with a fully known θ. We characterize this relationship between the demand learning time and the firm's optimal solution in the following proposition. It follows that when the demand learning time takes extreme values, the solution of the [TALP] reduces to one of the single-stage problems.

Proposition 1

(i) *There exists a threshold in the learning time $\bar{\tau}$ such that if $\bar{\tau} \leq T$, the optimal first stage solution of the* [*TALP*] (5) *coincides with the optimal solution of the* [*SALP-U*] (2).
(ii) *There exists a threshold in the learning time $\underline{\tau}$ such that if $\underline{\tau} \geq T$, the optimal second stage solution of the* [*TALP*] (5) *coincides with the optimal solution of the* [*SALP*] (1) *for a given θ.*

Proof Let us denote the first-stage expected profit per unit time by $\pi_1(X^1) = \mathbb{E}_\theta[\sum_{i\in I} r\theta h_i g_i(d_i(X^1)) - f(X^1)]$ and similarly the second-stage expected profit per unit time as $\pi_2(X^2; X^1) = \sum_{i\in I} r\theta h_i g_i(d_i(X^1 \cup X^2)) - f(X^1 \cup X^2)$, where X^2 is a policy $X^2(X^1, \theta)$ with the arguments suppressed for notational brevity. Then, the objective function can be written as follows: $Z_{[TALP]} = \frac{1}{\alpha}\{(1-e^{-\alpha T})\pi_1(X^1) + e^{-\alpha T}\pi_2(X^2; X^1)\}$.

(i) Let X^* be the optimal solution for the [SALP-U]. We show that there exists $\bar{\tau}$ such that, if $\bar{\tau} \le T$, then

$$\begin{aligned} & Z_{[TALP]}(X^*, X^2) \\ & \ge Z_{[TALP]}(\tilde{X}^1, \tilde{X}^2) \\ & \iff \quad \frac{1}{\alpha}\{(1-e^{-\alpha T})\pi_1(X^*) + e^{-\alpha T}\pi_2(X^2; X^*)\} \\ & \ge \frac{1}{\alpha}\{(1-e^{-\alpha T})\pi_1(\tilde{X}^1) + e^{-\alpha T}\pi_2(\tilde{X}^2; \tilde{X}^1)\} \\ & \iff \quad \pi_1(X^*) \ge \frac{e^{-\alpha T}}{(1-e^{-\alpha T})}[\pi_2(\tilde{X}^2; \tilde{X}^1) - \pi_2(X^2; X^*)] + \pi_1(\tilde{X}^1) \end{aligned} \tag{6}$$

holds for any $(\tilde{X}^1, \tilde{X}^2)$ and X^2. Since $\pi_1(X^*) \ge \pi_1(\tilde{X}^1)$ and $\frac{e^{-\alpha T}}{(1-e^{-\alpha T})}$ approaches to 0 as T increases, there exists $\bar{\tau}$ such that satisfies (6) if $\bar{\tau} \le T$.

(ii) Let X^* be the optimal solution for the [SALP] for a given θ. Similar to (i), we show that there exists $\underline{\tau}$ such that, if $\underline{\tau} \ge T$, then

$$\begin{aligned} & Z_{[TALP]}(X^1, X^*) \ge Z_{[TALP]}(\tilde{X}^1, \tilde{X}^2) \\ \iff \quad & \pi_2(X^*; X^1) \ge \frac{(1-e^{-\alpha T})}{e^{-\alpha T}}[\pi_1(\tilde{X}^1) - \pi_1(X^1)] + \pi_2(\tilde{X}^2; \tilde{X}^1) \end{aligned} \tag{7}$$

holds for any $(\tilde{X}^1, \tilde{X}^2)$ and X^1. Since $\frac{(1-e^{-\alpha T})}{e^{-\alpha T}}$ is increasing in T, the threshold value $\underline{\tau}$ which satisfies (7) can be obtained when $X^1 = \emptyset$. That is,

$$T \le \underline{\tau} = -\frac{1}{\alpha}\ln\left[\max\left[\frac{\pi_1(\tilde{X}^1)}{\pi_2(X^*; \emptyset) - \pi_2(\tilde{X}^2; \tilde{X}^1) + \pi_1(\tilde{X}^1)}\right]\right].$$

Note $\pi_2(X^*; \emptyset) \ge \pi_2(\tilde{X}^2; \tilde{X}^1)$ for any θ, thus $0 \le \frac{\pi_1(\tilde{X}^1)}{\pi_2(X^*;\emptyset) - \pi_2(\tilde{X}^2;\tilde{X}^1) + \pi_1(\tilde{X}^1)} \le 1$. Therefore, there exists $\underline{\tau}$ such that satisfies (7) if $T \le \underline{\tau}$. □

Proposition 1(i) implies that if the demand learning time is long enough, then the optimal first-stage action will be identical to the single-stage case (no reason to defer the commitment). This is because the second-stage profit will be small enough to not affect the first-stage decision. In contrast, Proposition 1(ii) implies that if the intrinsic demand learning time is fast enough, there is no incentive for the firm to take action in the first stage. The firm can rather maximize the profit by forgoing the first stage and deploy stations on the second stage with full demand information (extreme case of deferred commitment). To summarize, adding the dynamic nature (two-stage) to the problem generates incentive for the firm to delay the AFS deployment. In fact, the proposition suggests that the firm can always benefit by deferring the deployment of some stations in the second stage (in which the deployment becomes an option to be exercised depending on the realization of θ), unless T takes an extremely large value. As the demand learning time T decreases, the firm is less likely to be aggressive in the first stage.

We now consider that the demand learning time, T, is *endogenously* determined as a function of first-stage action. Although T can be a function of X^1 in *any* form, at this moment, we assume it depends on the "first-stage coverage," defined as $c(X^1) = \sum_{i\in I} h_i g_i(d(X^1))$. More specifically, we assume $T = \phi(c(X^1)) > 0$ is a decreasing function in $c(X^1)$ with some finite intrinsic learning time $\phi(\emptyset)$. Hence, the more potential trips are covered in the first stage, the faster the service demand rate is learned. In Sect. 5.1, we introduce a more specific learning time function for the numerical study. Finally, the Two-stage AFS Location Problem with Learning [TALP-L] is:

$$\textbf{[TALP-L]} \quad \max_{\substack{X^1\subset J \\ X^2(X^1,\theta)\subset J\setminus X^1, \forall X^1,\theta}} \frac{1}{\alpha}\left\{\mathbb{E}_\theta\left[\left(1-e^{-\alpha T}\right)\sum_{i\in I} r\theta h_i g_i\left(d_i\left(X^1\right)\right) - f\left(X^1\right)\right.\right.$$
$$\left.\left. + e^{-\alpha T}\left(\sum_{i\in I} r\theta h_i g_i\left(d_i\left(X^1\cup X^2\left(X^1,\theta\right)\right)\right) - f\left(X^2\left(X^1,\theta\right)\right)\right)\right]\right\} \tag{8}$$

where the learning time is $T = \phi(c(X^1))$. The endogenous learning time introduces incentive for the firm to actively learn the market by deploying facilities in the first stage. If the firm makes an aggressive investment in the first stage, the demand learning time shortens and thus the firm can expedite the time to have the fully known θ (and then deploy additional stations). We illustrate this effect by contrasting the optimal solution of the [TALP-L] to that of the [TALP] in the following proposition.

Proposition 2 *Let* $(X^{1*}, X^{2*}(X^{1*},\theta))$ *be the optimal solution of the* [*TALP-L*] (8) *and* $c(X^{1*})$, $T^* = \phi(c(X^{1*}))$ *be the corresponding first-stage coverage and the learning time. For the exogenous learning time* $T = T^*$, *let* $(X_T^{1*}, X_T^{2*}(X_T^{1*},\theta))$ *be the optimal solution of the* [*TALP*] *and* $c(X_T^{1*})$ *be the corresponding first-stage coverage. Then,* $c(X_T^{1*}) \le c(X^{1*})$.

Proof

Lemma 1 *The optimal objective value* $Z_{[TALP]}$ *is a decreasing function in* T.

Proof As in the proof of Proposition 1, we express the objective function of the [TALP] as $Z_{[TALP]} = \frac{1}{\alpha}\{(1-e^{-\alpha T})\pi_1(X^1) + e^{-\alpha T}\pi_2(X^2;X^1)\}$ where $\pi_1(X^1)$ and $\pi_2(X^2;X^1)$ are the first-stage and second-stage expected profit per unit time, respectively. Note that we suppress the arguments of the function X^2 for brevity. Thus, $Z_{[TALP]} = \frac{1}{\alpha}\{\pi_1(X^1) + e^{-\alpha T}(\pi_2(X^2;X^1) - \pi_1(X^1))\}$. Since $\pi_1(X^1) \le \pi_2(X^2;X^1)$, we know that $Z_{[TALP]}$ is decreasing in T.

First, we know $Z_{[TALP-L]}(X^{1*}, X^{2*}) \le Z_{[TALP]}(X_T^{1*}, X_T^{2*})$ if $T = T^*$ is given for the [TALP] (because [TALP] has less constraint than [TALP-L]). Now, suppose $c(X_T^{1*}) > c(X^{1*})$. This implies $T^* > T = \phi(c(X_T^{1*}))$ since $\phi(c)$ is decreasing in coverage c. From Lemma 1, we know that $\frac{1}{\alpha}\{(1-e^{-\alpha T^*})\pi_1(X_T^{1*}) + e^{-\alpha T^*}\pi_2(X_T^{2*})\} < \frac{1}{\alpha}\{(1-e^{-\alpha T})\pi_1(X_T^{1*}) + e^{-\alpha T^*}\pi_2(X_T^{2*})\}$.

Thus, it follows that

$$Z_{[TALP-L]}\left(X^{1*}, X^{2*}\right) = \frac{1}{\alpha}\left\{\left(1-e^{-\alpha T^*}\right)\pi_1\left(X^{1*}\right) + e^{-\alpha T^*}\pi_2\left(X^{2*}\right)\right\}$$

$$\leq \frac{1}{\alpha}\big\{\big(1-e^{-\alpha T^*}\big)\pi_1\big(X_T^{1*}\big)+e^{-\alpha T^*}\pi_2\big(X_T^{2*}\big)\big\}$$

$$< \frac{1}{\alpha}\big\{\big(1-e^{-\alpha T}\big)\pi_1\big(X_T^{1*}\big)+e^{-\alpha T}\pi_2\big(X_T^{2*}\big)\big\} = Z_{[TALP-L]}\big(X_T^{1*}, X_T^{2*}\big).$$

This is a contradiction since $Z_{TALP-L}(X^{1*}, X^{2*})$ is the optimal objective value for the [TALP-L]. □

The proposition shows that the presence of endogenous learning allows the AFS service provider to learn the service demand rate faster by covering more potential trips in the first stage. This is because the decision maker for the [TALP] does not have any incentive to aggressively deploy stations if the demand learning time is exogenously given.

As shown in Proposition 1 and 2, the proposed two-stage location problem illustrates the trade-off between active learning and deferred commitment in AFS deployment. While the service provider always benefits from a short learning time, the first-stage decision is irreversible in the future. Hence, a short-sighted initial decision may lead to sub-optimal station locations and harm the firm in the long run.

3.3 Nonlinear integer programming formulation

Before we propose a solution method for the [TALP-L], we first reformulate the problem as a nonlinear integer programming problem. Let θ be a discrete random variable with $|S|$ possible outcomes (scenarios) such that the probability of a scenario s is $P(\theta=\theta^s)=p^s$ and $\sum_{s\in S} p^s = 1$. Thus, the problem can be formulated as follows:

[P1]

$$\max_{\mathbf{X},\mathbf{Y},T} \sum_{s\in S}\frac{p^s}{\alpha}\Bigg[\big(1-e^{-\alpha T}\big)\sum_{i\in I}\sum_{j\in J} r\theta^s h_i g_i(d_{ij})Y_{ij}^1 - \sum_{j\in J} f_j X_j^1 + e^{-\alpha T}\Bigg[\sum_{i\in I}\sum_{j\in J} r\theta^s h_i g_i(d_{ij})Y_{ijs}^2 - \sum_{j\in J} f_j X_{js}^2\Bigg]\Bigg] \quad (9)$$

$$\text{s.t.}\quad Y_{ij}^1 \leq X_j^1, \quad Y_{ijs}^2 \leq X_j^1 + X_{js}^2 \quad \forall i\in I, \forall j \in J, \forall s\in S, \quad (10)$$

$$\sum_{j\in J} Y_{ij}^1 = 1, \quad \sum_{j\in J} Y_{ijs}^2 = 1 \quad \forall i \in I, \forall s\in S, \quad (11)$$

$$X_j^1 + X_{js}^2 \leq 1 \quad \forall j\in J, \forall s \in S, \quad (12)$$

$$T = \phi\Bigg(\sum_{i\in I}\sum_{j\in J} h_i g_i(d_{ij})Y_{ij}^1\Bigg), \quad (13)$$

$$X_j^1, X_{js}^2, Y_{ij}^1, Y_{ijs}^2 \in \{0,1\} \quad \forall i\in I, \forall j\in J, \forall s\in S. \quad (14)$$

The objective function (9) consists of four terms. The first term represents the expected present value discounting the revenue over T periods of time for Stage 1 and the second term accounts for the total cost of the operating AFS opened in the first stage over the two stages. The third term represents the discounted value of the second-stage revenue and the last term represents the total cost for the operating AFS opened in the second stage discounted at present value from time T to ∞. The constraints (10) state that each attraction node can be covered only by an open station for each stage. The constraints (11) ensure that each attraction node is covered by at least one station. Constraints (12) state that we cannot locate another station at the same location if one already exists. We refer to the constraint (13) as the Coverage constraint, since it expresses the relationship between the learning time T and the coverage $c(X^1, Y^1) = \sum_{i \in I} \sum_{j \in J} h_i g_i(d_{ij}) Y^1_{ij}$. Finally, the constraints (14) represent the non-negativity and integrality of the decision variables.

The above problem is a mixed integer program with X^1, Y^1, X^2, and Y^2 as decision variables and T as an auxiliary decision variable. The learning time T is endogenously determined by the first-stage demand coverage, defined as $c(X^1, Y^1)$. For convenience, we will use c to represent the coverage in the first stage and also use $T = \phi(c)$. We obtain the upper bound in coverage, $\overline{c}$, by opening all the AFS in the first stage. Also, we obtain a lower bound in coverage, $\underline{c}$, by finding the commonly opened AFS by solving the single-stage problem, [SALP], for each scenario. Since the endogeneity of the learning time T brings nonlinearity to the objective function (9), the currently proposed formulation [P1] is very challenging to solve. In the next section, we propose a solution method for this problem.

4 Solution approach

4.1 Solution method

In this section, we develop an approximate solution method to solve the proposed nonlinear integer program. Note that the feasible region of [P1] may not necessarily be a convex set due to (13). Mahajan and Munson (2010) proposed to solve a class of nonlinear programming problems involving non-convex feasible regions by decomposing the feasible regions into several convex sets. Similar to this approach, we decompose the feasible region into several subproblems with convex feasible region and then use a standard convex optimization technique to solve the individual subproblems.

To remove the exponential terms in the objective function in [P1], we first introduce a new decision variable $W = e^{-\alpha T}$. Then, the following holds.

Lemma 2 *$W = e^{-\alpha T}$ is an increasing function of the coverage $c(X^1, Y^1)$.*

Proof Since $T = \phi(X^1, Y^1) = \phi(c(X^1, Y^1))$ is a decreasing function of coverage $c(X^1, Y^1)$, so T decreases as we increase the coverage and $W = e^{-\alpha T}$ being a decreasing function of T increases with a decrease in T. Hence $W(c) = e^{\phi(c)} = e^{-\alpha T}$ is an increasing function of coverage. □

With the new variable W, the constraint (13) can be replaced with $W \leq e^{-\alpha\phi(c)}$, which eliminates the decision variable T. Since this constraint may create non-convexity in the feasible region, we approximate $W = e^{-\alpha\phi(c)}$ by $\widehat{W}$ using the piecewise linear functions of c. More specifically, we separate the range of the first-stage coverage into a number of intervals such that the linear approximation of W in each interval satisfies $0 \leq \frac{W - \widehat{W}}{W} \leq \epsilon$. The error rate ϵ determines the precision level of the proposed approximation. Denoting the resulting intervals by $k \in K$, we represent the lower and upper bounds of the coverage for each interval by $\underline{c}_k$ and $\overline{c}_k$, and the corresponding bounds of W by $\underline{\omega}_k$ and $\overline{\omega}_k$, respectively. The linear approximation in the kth interval can be expressed as $\widehat{W} = a_k + b_k c_k$, where a_k and b_k are constants, and c_k denotes the coverage $c(X^1, Y^1)$ restricted in the subinterval k i.e., $c_k \in [\underline{c}_k, \overline{c}_k]$. Recall that, by Lemma 2, the learning time T is a decreasing function of coverage. This implies that the piecewise linear approximation of $W = e^{-\alpha\phi(c)}$ should be an increasing function of c, and thus the intervals $[\underline{\omega}_k, \overline{\omega}_k]$ are non-overlapping.

We approximate the coverage constraint by $|K|$ linear pieces with the domain restricted to $[\underline{c}_k, \overline{c}_k]$ for the kth approximation. Hence, the [P1] is decomposed into $|K|$ subproblems. The optimal solution of [P1] then corresponds to the maximum of the optimal solutions of these subproblems. Hence, the [P1] assumes the following form:

$$\text{[P2]} \quad \max_{k\in K} \Pi_k, \quad \text{where}$$

$$\begin{aligned}
\Pi_k = \max_{\mathbf{X},\mathbf{Y},\widehat{W}} \sum_{s\in S} \frac{p^s}{\alpha}\bigg[(1-\widehat{W})\sum_{i\in I}\sum_{j\in J} r\theta^s h_i g_i(d_{ij})Y^1_{ij} - \sum_{j\in J} f_j X^1_j \\
+ \widehat{W}\bigg[\sum_{i\in I}\sum_{j\in J} r\theta^s h_i g_i(d_{ij})Y^2_{ijs} - \sum_{j\in J} f_j X^2_{js}\bigg]\bigg]
\end{aligned}$$

$$\begin{aligned}
\text{s.t.}\quad & Y^1_{ij} \le X^1_j, \quad Y^2_{ijs} \le X^1_j + X^2_{js} \quad \forall i\in I, \forall j\in J, \forall s\in S,\\
& \sum_{j\in J} Y^1_{ij} = 1, \quad \sum_{j\in J} Y^2_{ijs} = 1 \quad \forall i\in I, \forall s\in S,\\
& X^1_j + X^2_{js} \le 1 \quad \forall j\in J, \forall s\in S,\\
& \widehat{W} = a_k + b_k c_k,\\
& \widehat{W} \in [\underline{\omega}_k, \overline{\omega}_k], \quad X^1_j, X^2_{js}, Y^1_{ij}, Y^2_{ijs} \in \{0,1\} \quad \forall i\in I, \forall j\in J, \forall s\in S.
\end{aligned}$$

The [P2] is an approximation of the [P1]; however, we can find the bound on the relative error that accumulates in approximating the total profit over the two stages.

Proposition 3 *Let $\Pi^*(W)$ and $\Pi^*(\widehat{W})$ be the optimal profits of the problems [P1] and [P2], respectively. Then, $\Pi^*(\widehat{W})$ is a lower bound for $\Pi^*(W)$ and the relative error between $\Pi^*(\widehat{W})$ and $\Pi^*(W)$ is bounded by the relative error rate of the linear approximation, ϵ; i.e., $\frac{\Pi^*(W)-\Pi^*(\widehat{W})}{\Pi^*(W)} \le \epsilon$.*

Proof Let us denote the first stage expected profit per unit time (terms in the objective function corresponding to the first stage) by $\pi_1(X^1, Y^1) = \sum_s p^s[\sum_i \sum_j r\theta^s h_i g_i(d_{ij})Y^1_{ij} - \sum_j f_j X^1_j]$ and the second-stage expected profit per unit time as $\pi_2(X^2, Y^2; X^1, Y^1) = \sum_s \sum_i \sum_j rp^s\theta^s h_i g_i(d_{ij})Y^2_{ijs} - \sum_s \sum_j p^s f_j X^2_{js} - \sum_j f_j X^1_j$. Therefore, the optimal objective function can be expressed as:

$$\begin{aligned}
\Pi^*(W) &= \frac{1}{\alpha}\big[(1-W)\pi_1\big(X^{1*}, Y^{1*}\big) + W\pi_2\big(X^{2*}, Y^{2*}; X^{1*}, Y^{1*}\big)\big]\\
&= \frac{1}{\alpha}\big[\pi_1\big(X^{1*}, Y^{1*}\big) + W\big(\pi_2\big(X^{2*}, Y^{2*}; X^{1*}, Y^{1*}\big) - \pi_1\big(X^{1*}, Y^{1*}\big)\big)\big]\\
&= \frac{1}{\alpha}\big[\pi^*_1 + W\big(\pi^*_2 - \pi^*_1\big)\big].
\end{aligned}$$

Note that $\pi_1(X^{1*}, Y^{1*}) \le \pi_2(X^{2*}, Y^{2*}; X^{1*}, Y^{1*})$ holds since one can only improve the expected unit profit in the second stage by deploying additional stations (otherwise, one can preserve the first-stage expected unit profit by choosing not to open new stations).

Now from the error rate inequality $\frac{W-\widehat{W}}{W} \le \epsilon$, it follows that $(1-\epsilon)W \le \widehat{W}$ and thus

$$\begin{aligned}(1-\epsilon)\Pi^*(W) &= (1-\epsilon)\frac{1}{\alpha}\big[\pi_1^* + W\big(\pi_2^* - \pi_1^*\big)\big] \\ &\le \frac{1}{\alpha}\big[\pi_1^* + (1-\epsilon)W\big(\pi_2^* - \pi_1^*\big)\big] \\ &\le \frac{1}{\alpha}\big[\pi_1^* + \widehat{W}\big(\pi_2^* - \pi_1^*\big)\big] = \Pi^*(\widehat{W}).\end{aligned}$$

Hence, we have $\frac{\Pi^*(W)-\Pi^*(\widehat{W})}{\Pi^*(W)} \le \epsilon$. □

Proposition 3 states that the approximate profit obtained by solving the [P2] provides a good lower bound on the true optimal profit which is the objective value of the [P1]. This enables us to bound the relative error rate in the approximation and the true profit by ϵ. Since this error rate is the same as the error rate of the linear approximation, the AFS network designer can achieve the desired precision in the profit approximation by appropriately choosing the error rate ϵ. These results are based on the fact that we intend to under-approximate the profit; thus, it provides us with the lowest amount of profit that can be obtained following the approximation. In other words, the decision maker can be conservative while approximating the value of W, so that the approximated profit provides a lower bound for the exact solution.

Although the constraint sets in [P2] are linear, the objective functions Π_k's still contain nonlinear terms involving products of decision variables W with Y^1, Y^2 and X^2. We use the technique of Oral and Kettani (1992) to linearize the objective function by exploiting the fact that Y^1, Y^2 and X^2 are binary variables. We first define the coefficients of these binary variables as $D_{ij}^{Y1}(\widehat{W}) = (\frac{1}{\alpha})rh_i g_i(d_{ij})\sum_{s\in S} p^s\theta^s\widehat{W}$, $D_{ijs}^{Y2}(\widehat{W}) = (\frac{1}{\alpha})rp^s\theta^s h_i g_i(d_{ij})\widehat{W}$, and $D_{js}^{X2}(\widehat{W}) = (\frac{1}{\alpha})p^s f_j\widehat{W}, \forall i \in I, \forall j \in J, \forall s \in S$ and express the objective functions as

$$\begin{aligned}\max_{\mathbf{X},\mathbf{Y},\widehat{W}} &\sum_{i\in I}\sum_{j\in J} D_{ij}^{Y1}(1)Y_{ij}^1 - \sum_{i\in I}\sum_{j\in J} D_{ij}^{Y1}(\widehat{W})Y_{ij}^1 - \frac{1}{\alpha}\sum_{s\in S} p^s \sum_{j\in J} f_j X_j^1 \\ &+ \sum_{s\in S}\sum_{i\in I}\sum_{j\in J} D_{ijs}^{Y2}(\widehat{W})Y_{ijs}^2 - \sum_{s\in S}\sum_{j\in J} D_{js}^{X2}(\widehat{W})X_{js}^2. \end{aligned} \tag{15}$$

The coefficient functions $D's$ are increasing functions of $\widehat{W}$ and consequently achieve the following lower and upper bounds at the respective bounds of $\widehat{W}$.

$$\begin{aligned}\underline{D}_{ij}^{Y1} &\le D_{ij}^{Y1}(\widehat{W}) \le \overline{D}_{ij}^{Y1} \\ \underline{D}_{ijs}^{Y2} &\le D_{ijs}^{Y2}(\widehat{W}) \le \overline{D}_{ijs}^{Y2} \\ \underline{D}_{js}^{X2} &\le D_{js}^{X2}(\widehat{W}) \le \overline{D}_{js}^{X2}\end{aligned}$$

Now corresponding to each binary variable Y^1, Y^2 and X^2 we introduce continuous decision variables ζ^{Y1}, ζ^{Y2} and ζ^{X2} and a constraint respectively. We finally express the [P2] as follows:

[P3] $\max_{k\in K} \Pi_k$, where

$$\Pi_k = \max_{\mathbf{X},\mathbf{Y},\widehat{W},\zeta_{ij}^{Y1},\zeta_{ijs}^{Y2},\zeta_{js}^{X2}} \sum_{i\in I}\sum_{j\in J} D_{ij}^{Y1}(1)Y_{ij}^1 - \sum_{i\in I}\sum_{j\in J}\left(\underline{D}_{ij}^{Y1}Y_{ij}^1 + \zeta_{ij}^{Y1}\right) - \frac{1}{\alpha}\sum_{s\in S} p^s \sum_{j\in J} f_j X_j^1$$

$$+ \sum_{s\in S}\sum_{i\in I}\sum_{j\in J}\left(\overline{D}_{ijs}^{Y2}Y_{ijs}^2 - \zeta_{ijs}^{Y2}\right) - \sum_{s\in S}\sum_{j\in J}\left(\underline{D}_{js}^{X2}X_{js}^2 + \zeta_{js}^{X2}\right)$$

$$\text{s.t.} \quad \zeta_{ij}^{Y1} \ge D_{ij}^{Y1}(\widehat{W}) + \left(\overline{D}_{ij}^{Y1} - \underline{D}_{ij}^{Y1}\right)Y_{ij}^1 - \overline{D}_{ij}^{Y1} \quad \forall i \in I, \forall j \in J,$$

$$\zeta_{ijs}^{Y2} \ge -D_{ijs}^{Y2}(\widehat{W}) + \left(\overline{D}_{ijs}^{Y2} - \underline{D}_{ijs}^{Y2}\right)Y_{ij}^2 + \underline{D}_{ijs}^{Y2} \quad \forall i \in I, \forall j \in J, \forall s \in S,$$

$$\zeta_{js}^{X2} \ge D_{js}^{X2}(\widehat{W}) + \left(\overline{D}_{js}^{X2} - \underline{D}_{js}^{X2}\right)X_{js}^2 - \overline{D}_{js}^{X2} \quad \forall j \in J, \forall s \in S,$$

$$Y_{ij}^1 \le X_j^1, \quad Y_{ijs}^2 \le X_j^1 + X_{js}^2 \quad \forall i \in I, \forall j \in J, \forall s \in S,$$

$$\sum_{j\in J} Y_{ij}^1 = 1, \quad \sum_{j\in J} Y_{ijs}^2 = 1 \quad \forall i \in I, \forall s \in S,$$

$$X_j^1 + X_{js}^2 \le 1 \quad \forall j \in J, \forall s \in S,$$

$$\widehat{W} = a_k + b_k c_k,$$

$$X_j^1, X_{js}^2, Y_{ij}^1, Y_{ijs}^2 \in \{0,1\}, \quad \zeta_{ij}^{Y1}, \zeta_{ijs}^{Y2}, \zeta_{js}^{X2} \ge 0,$$

$$\widehat{W} \in [\underline{\omega}_k, \overline{\omega}_k] \quad \forall i \in I, \forall j \in J, \forall s \in S.$$

The newly introduced constraints corresponding to each binary variable allow a certain term of the objective function to act solely as a function of $\widehat{W}$ if the binary variable in the term is 1; and forces the term to be 0 otherwise [for detailed proof see Oral and Kettani (1992)], thus making [P3] equivalent to [P2].

This is a Mixed Linear Integer Program [MILP], which can be solved using commercial solvers such as CPLEX. In the next section, we show that the computational performance with realistic-sized instances is quite satisfactory.

4.2 Computational performance

We now conduct a set of numerical experiments to illustrate the performance of the proposed solution method. We use the 2000 census data from the City of Chicago (Census.gov 2010) to generate two networks with 43 and 102 demand nodes. The data include the distance matrix with each element representing the distance between the nodes d_{ij}, the average income of a household, and the population at various nodes. We assume each census tract represents one attraction node $i \in I$. We generate the facility cost f_i proportional to the average income (to reflect the land price) and the potential AFV users h_i by its population. Specific values of other parameters are provided in the numerical results in Sect. 5. The algorithm was coded in C++ by integrating ILOG CPLEX 12.2, which was run on an HP Z400 desktop with a 2.93 GHz CPU and 16 GB of RAM.

By varying the size of the scenarios and the error rate ϵ, it was shown that the proposed algorithm solves the problem in a reasonable time with sufficient accuracy. We use networks of size 43 nodes (downtown Chicago) and 102 nodes (expanded Chicago downtown) with three different sets of AFS candidate sites to evaluate the performance of the proposed algorithm. For each of these settings, we generate 10 instances by varying the distribution of adoption rate θ. In particular, we assume that θ is uniformly distributed with mean 0.5 for all instances, and vary its standard deviation from 0.03 to 0.30 to generate different instances.

Table 1 Computation times (in seconds) of the proposed algorithm

Demand ($\|I\|$)	Candidate Sites ($\|J\|$)	Scenarios ($\|S\|$)			
		$\epsilon = 0.1\%$		$\epsilon = 0.01\%$	
		25	50	25	50
43	10	19	22	40	44
	20	31	44	58	83
	30	190	311	380	594
102	30	216	681	402	1,239
	50	823	2,991	1,541	5,429
	70	1,507	9,523	2,703	15,023

We report the average solution time of these 10 instances for both 25 and 50 scenarios. We use the demand learning time function $T = \frac{\sigma^2}{\gamma c(X^1, Y^1)}$ with all parameters identical to Sect. 5. The solution times are presented for the two levels of error rate, $\epsilon = 0.1$ % and 0.01%. The computational performance of the algorithm (solution time measured in seconds) is illustrated in Table 1.

As shown in the Table 1, the proposed algorithm solves the problem efficiently. The largest instance in the study with 102 demand nodes and 70 AFS candidate locations was solved in approximately 9,500 seconds for the $\epsilon = 0.1\%$ case and in 15,000 seconds for the $\epsilon = 0.01\%$ case on average. Note that there is a significant difference between the solution times of the 25 and 50 scenarios, suggesting that the number of scenarios is a key determinant of solution time.

5 Numerical study

In this section, we numerically study the impact of demand learning on the AFS locations and derive relevant managerial insights. In Sect. 5.1, we first characterize the service provider's optimal AFS deployment strategy under different market characteristics. In Sect. 5.1, we study the value of having foresight in the presence of demand learning effect. Throughout the section, we use a numerical example of a network with 102 attraction nodes and 70 candidate AFS locations with 10 scenarios.

5.1 Optimal AFS deployment strategy

We characterize the market condition of the AFS service region by considering different learning time functions. In particular, we consider different levels of (i) speed of learning (how fast the market can be learned, captured by a coefficient $\gamma > 0$) and (ii) market variability (degree of uncertainty in the demand rate θ, captured by the variance of demand rate σ^2). To numerically study the impact of demand learning effect, we employ a specific demand learning time function defined as $T = \frac{\sigma^2}{\gamma c(X^1, Y^1)}$, where $c(X^1, Y^1)$ is the first-stage coverage, i.e., $\sum_i \sum_j \theta h_i g_i(d_{ij}(Y_{ij}^1))$. Here, higher γ values represent faster demand learning, which leads to decreasing demand learning time T. In contrast, higher degrees of σ results in slower demand learning. Although we employ a specific form of T in this section, we note that our solution approach is applicable to any form of a learning time function.

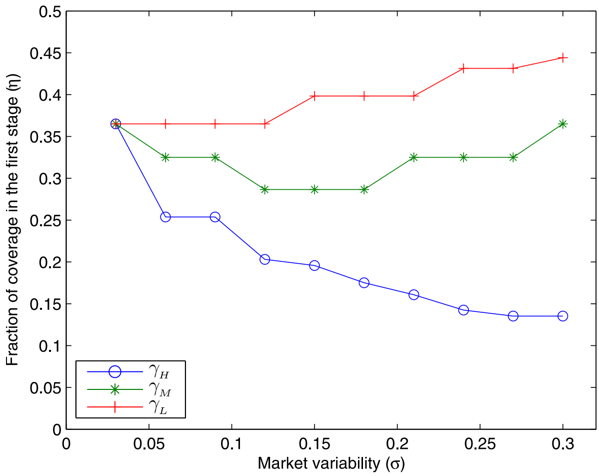


Fig. 1 Optimal AFS deployment strategy under different market characteristics

For the numerical example, we assume that the demand rate θ follows a uniform distribution with mean of 0.5 and the standard deviation was varied by changing the support of the distribution. We obtained 10 scenarios by varying σ from 0.03 to 0.3 with increments of 0.03. We set the remaining parameters to $\alpha = 0.05$, $\lambda = 0.01$, $r = 190$ with an error rate of $\epsilon = 0.001(0.1\ \%)$. A unit of time is set equal to a quarter of a year. For convenience, we define the fraction of the service demand coverage in the first stage as $\eta = \frac{c(X^1, Y^1)}{\bar{c}}$, where $\bar{c}$ is the upper bound of the demand coverage (i.e., maximum coverage achieved when all facilities are opened). This measure captures the firm's optimal deployment strategy; i.e., an increasing η suggests that a firm prefers active learning, whereas a decreasing η suggests that a firm prefers deferred commitment.

Figure 1 illustrates the three different types of the firm's optimal deployment strategies for the three different levels of γ: $\gamma_H = 50 \times 10^5$ (fast-learning market), $\gamma_M = 3 \times 10^5$ (moderate market), and $\gamma_L = 0.8 \times 10^5$ (slow-learning market). We observe that η decreases with σ for γ_H but increases for γ_L. Hence, for the fast-learning markets, deferred commitment in AFS deployment (open less AFS in the first stage) is recommended as market variability increases. This because the benefit of actively learning the market (expediting the demand learning time) is limited when the learning speed of the market is fast (T is relatively short even if the firm does not cover much in the first stage). For the slow-learning markets, however, we recommend early deployment to actively learn the market (open more AFS in the first stage) as market variability increases. This is because the firm attempts to compensate for the slow learning speed by increasing the demand coverage. For the moderate market (γ_M), we observe that η first decreases and then increases with the increase in σ. That is, the firm first prefers the deferred commitment strategy but later prefers the active learning strategy as uncertainty in the market increases. This result reveals that the speed of market learning and the market variability are interacting; the firm's optimal deployment decision is influenced more by the learning speed when σ is small, but is influenced more by the market variability as σ becomes sufficiently large.

Table 2 compares the fraction of the demand coverage in the first stage η, the expected total profit Π, and the demand learning time T for the various values of γ and σ. We observe that Π increases as γ increases for a fixed σ since the demand learning time T decreases with γ. We also observe that Π increases with σ (except for the case when σ is very high and γ is very low). This reveals that a firm typically gains a higher expected total profit

Table 2 Fraction of demand coverage in the first stage, expected total profits (in thousand $) and demand learning time (in quarter of a year) under different market characteristics

		$\sigma = 0.03$	0.06	0.09	0.12	0.16	0.18	0.21	0.24	0.27	0.30
Hi	η	0.3650	0.2537	0.2537	0.2031	0.1958	0.1751	0.1607	0.1425	0.1352	0.1352
(γ_H)	Π	18,056	18,640	19,554	20,712	22,055	23,491	25,099	26,518	28,201	29,722
	T	0.0070	0.0400	0.0901	0.2001	0.3242	0.5220	0.7744	1.1409	1.5219	1.8789
Mod.	η	0.3650	0.3251	0.3251	0.2865	0.2865	0.2865	0.3251	0.3251	0.3251	0.3650
(γ_M)	Π	18,053	18,577	19,253	20,031	20,755	21,345	21,826	22,190	22,428	22,569
	T	0.1113	0.5000	1.1251	2.2692	3.5456	5.1057	6.1254	8.0006	10.126	11.133
Low	η	0.3650	0.3650	0.3650	0.3650	0.3984	0.3984	0.3984	0.4315	0.4315	0.4442
(γ_L)	Π	18,041	18,468	18,890	19,224	19,358	19,424	19,350	19,161	18,994	18,808
	T	0.4453	1.7812	4.0078	7.1249	10.201	14.689	19.994	24.111	30.515	36.590

Table 3 Impact of discount rate in the expected total profits (in thousand $)

		$\sigma = 0.03$	0.06	0.09	0.12	0.16	0.18	0.21	0.24	0.27	0.30
Hi	α_L	90,268	93,196	97,820	103,996	111,781	120,451	129,659	139,171	148,964	158,946
(γ_H)	α_M	18,057	18,640	19,555	20,712	22,055	23,491	25,099	26,518	28,201	29,722
	α_H	9,025	9,311	9,759	10,313	10,952	11,612	12,289	12,940	13,580	14,211
Mod.	α_L	90,235	93,120	97,511	102,812	108,680	114,686	120,629	126,242	131,613	136,838
(γ_M)	α_M	18,053	18,577	19,253	20,031	20,755	21,345	21,826	22,190	22,428	22,569
	α_H	9,022	9,264	9,555	9,831	10,057	10,203	10,296	10,313	10,279	10,224
Low	α_L	90,268	92,942	96,587	100,763	104,773	108,196	111,093	113,268	115,234	116,507
(γ_L)	α_M	18,041	18,468	18,890	19,224	19,358	19,424	19,350	19,161	18,994	18,808
	α_H	9,006	9,185	9,302	9,358	9,339	9,275	9,203	9,125	9,057	9,005

when entering a market with a higher uncertainty since it can take advantage of the higher demand rates in such markets with learning and recourse action. This insight coincides with the real options literature (Dixit and Pindyck 1994): the value of the real options increase as the market variability increases (the decision maker can exercise the option when the market turns out to be good; otherwise, the decision maker can simply stay put). When the learning speed is slow, however, the expected total profit starts to decrease as the market variability increases sufficiently high. This is because the demand learning time (i.e., the length of the first stage) becomes too long and the profit streams in the second stage are discounted too much. We finally note that the demand learning time increases as the market variability increases.

Finally, we study the impact of discount rate in Table 3 by varying the degrees of α ($\alpha_L = 0.01, \alpha_M = 0.05$, and $\alpha_H = 0.1$) under different levels of σ and γ. The remaining parameters are identical to the previous setting. As expected, lower degrees of discount rate lead to greater level of expected total profits. In addition, we find that, although the expected total profit generally increases with the degree of market uncertainty (σ), interestingly, high degree of discount rates coupled with low speed of market learning result in decrease in the expected total profit with market uncertainty. This implies that high discount rate depreciates the value of second stage significantly and thus eliminates the real option effect introduced earlier.

5.2 Value of foresight in the presence of demand learning

We now investigate the value of having foresight in planning the deployment of AFS in the presence of demand learning. As a benchmark, we consider a myopic planner who maximizes the profit for the current stage only without considering the recourse option. The myopic planner represents a decision maker without foresight; a service provider that focuses on the short-term profit and disregards the long-term plan with a pressure to perform in the immediate future. This exercise illustrates the value of the proposed model in relation to the other existing models in the literature that do not include the demand learning and recourse effect.

Similar to the optimal planner, we assume that the myopic planner knows the distribution of the demand rate and deploys stations to maximize the expected profit in the first stage. While making the first-stage decision, however, the myopic planner does not take into account the demand learning effect, although the learning process is still in place. Once the first stage elapses (after the learning is completed based on its first-stage coverage), the myopic planner is given the true demand rate and deploys the additional stations needed to maximize the total profit in the second stage. The formulation of the two-stage myopic AFS location problem is as follows:

$$\int_0^T e^{-\alpha t}\overline{V}\big(X^1\big)dt + \int_T^\infty e^{-\alpha t}\overline{V}\big(X^2; X^1, \theta\big)dt, \tag{16}$$

$$\text{where}\quad \overline{V}\big(X^1\big) = \max_{X^1\subset J}\left\{\mathbb{E}_\theta\left[\sum_{i\in I} r\theta h_i g_i\big(d_i\big(X^1\big)\big) - f\big(X^1\big)\right]\right\}$$

$$\text{and}\quad \overline{V}\big(X^2; X^1, \theta\big) = \max_{X^2\subset J\setminus X^1}\left\{\sum_{i\in I} r\theta h_i g_i\big(d_i\big(X^1\cup X^2\big)\big) - f\big(X^1\cup X^2\big)\right\}$$

$$\text{where } T = \phi\big(c\big(X^1\big), \sigma\big).$$

The objective function of the myopic planner's problem (16) can be rewritten as

$$\frac{1}{\alpha}\big\{\big(1-e^{-\alpha T}\big)\overline{V}\big(X^1\big) + e^{-\alpha T}\overline{V}\big(X^2; X^1, \theta\big)\big\}. \tag{17}$$

To contrast the behavior of the two service providers, we use the numerical examples with a high and low learning speed (γ_H, γ_L) introduced in Sect. 5.1. We let N_O, r_O, f_O, Π_O, and T_O, and N_M, r_M, f_M, Π_M, and T_M be the average number of opened stations, the total revenue, the total AFS operation cost, the expected total profit, and the resulting demand learning time that corresponds to the optimal and the myopic planner, respectively.

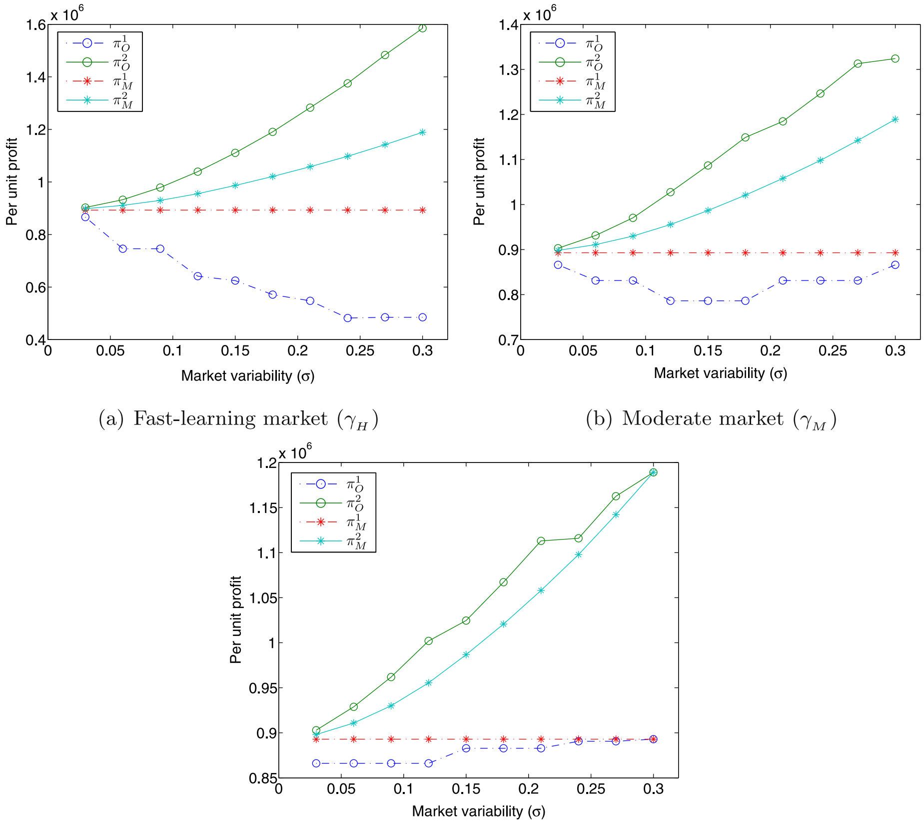
Table 4 compares the two planners' optimal decisions (N, r, f, Π, T) for the different levels of γ and σ. We observe that the difference in the optimal decisions increases with σ for γ_H, but decreases with σ for γ_L. Thus, in the presence of demand learning, the lack of foresight harms the myopic planner more under such conditions. In other words, the value of deferred commitment becomes greater as market variability increases in the fast-learning markets. However, the value of active learning becomes greater as market variability decreases in the slow-learning markets.

Table 4 Comparison of decisions for the optimal and the myopic planner (total profits are in thousand $)

		$\sigma = 0.03$	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
Hi (γ_H)	[**Optimal Planner**]										
	N_O	16.8	16.9	16.8	16.8	16.8	17.0	16.7	17.4	17.5	17.8
	r_O	59,325	60,252	60,833	62,675	64,376	64,924	66,479	67,831	70,269	72,274
	f_O	41,269	41,612	41,278	41,968	42,322	41,434	41,381	41,314	42,068	42,553
	Π_O	18,057	18,640	19,555	20,707	22,054	23,490	25,098	26,517	28,201	29,722
	T_O	0.0070	0.0400	0.0901	0.2001	0.3243	0.5220	0.7744	1.1409	1.5219	1.8789
	[**Myopic Planner**]										
	N_M	17.7	18.6	19.1	19.9	20.2	20.5	21.0	21.4	21.9	22.4
	r_M	62,022	65,231	67,161	70,956	72,398	74,002	76,187	77,813	80,450	83,127
	f_M	44,063	47,015	48,565	51,853	52,677	53,615	55,074	55,931	57,721	59,512
	Π_M	17,959	18,216	18,596	19,102	19,721	20,387	21,113	21,882	22,729	23,616
	T_M	0.0057	0.0229	0.0515	0.0915	0.1429	0.2059	0.2801	0.3659	0.4631	0.5717
Low (γ_L)	[**Optimal Planner**]										
	N_O	16.8	17.5	17.8	18.5	19.2	19.5	20.0	20.9	21.4	22.4
	r_O	59,105	60,867	60,977	61,853	63,619	62,492	61,329	63,961	62,972	63,669
	f_O	41,064	42,400	42,087	42,628	44,261	43,068	41,979	44,800	43,977	44,861
	Π_O	18,041	18,468	18,890	19,224	19,358	19,424	19,349	19,161	18,994	18,808
	T_O	0.4453	1.7812	4.0078	7.1249	10.201	14.689	19.994	24.111	30.515	36.590
	[**Myopic Planner**]										
	N_M	17.7	18.6	19.1	19.9	20.2	20.5	21.0	21.4	21.9	22.4
	r_M	61,982	64,855	66,063	68,168	67,838	67,236	66,592	65,502	64,616	63,669
	f_M	44,025	46,664	47,577	49,377	48,793	48,056	47,388	46,374	45,627	44,861
	Π_M	17,957	18,191	18,485	18,791	19,045	19,180	19,204	19,128	18,989	18,808
	T_M	0.3659	1.4636	3.2931	5.8544	9.1475	13.172	17.929	23.418	29.638	36.590

We further investigate the difference in the expected total profits by analyzing the per unit profit for each stage. Since the myopic planner seeks to maximize only the current stage's expected profit, its first-stage expected unit profit is always greater than or equal to that of the optimal planner's ($\pi_O^1 \le \pi_M^1$), as shown in Fig. 2. In the first stage, the optimal planner attempts to hedge against the market variability, thus the first-stage profit decreases as market variability increases. In the second stage, however, the optimal planner typically gains a greater expected unit profit ($\pi_O^2 \le \pi_M^2$). The first-stage expected unit profit for the myopic planner remains flat regardless of market variability because it does not take into account the second-stage recourse option.

Interestingly, the optimal planner tends to gain less in the first stage, but more in the second stage as σ increases for a high γ (Fig. 2(a)). However, the optimal planner tends to behave similar to the myopic planner as σ increases for a low γ (Fig. 2(c)). This effect is attributable to the stiff discount of the second-stage profits with the extended demand learning time (hence, the second stage becomes less relevant when making the first-stage decision). For the moderate level of γ, both trends were observed in each of the extreme values of σ (Fig. 2(b)).



(a) Fast-learning market (γ_H)

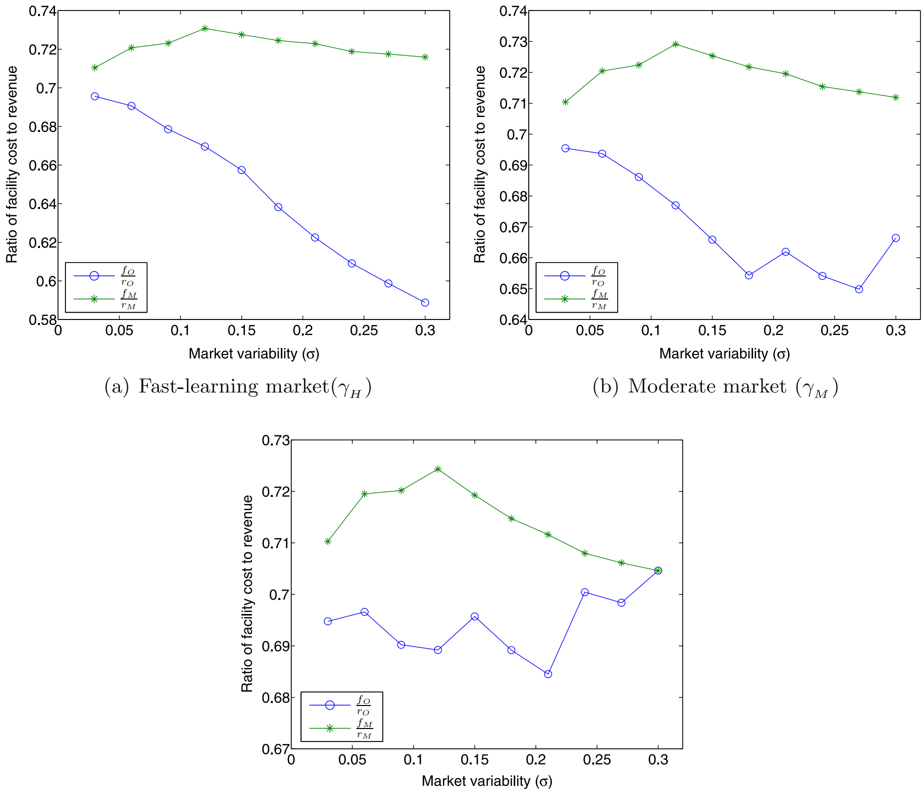
(b) Moderate market (γ_M)

(c) Slow-learning market (γ_L)

Fig. 2 Profit comparison for the optimal and the myopic planner at each stage

From Table 4, we also observe that the myopic planner invests more on the stations (both in terms of N and f) relative to the optimal planner due to the lack of foresight. Since the myopic planner invests more in the facility costs, the myopic planner typically gains a higher revenue than the optimal planner. However, the difference in the revenue and the facility cost (i.e., the expected total profit) of the optimal planner is always greater than that of the myopic planner, and this gap increases with market variability.

To better understand the over-commitment in facility investment for the myopic planner, we compare the ratio of total facility (station) operation cost to the total revenue for both planners, $\frac{f_O}{r_O}$ and $\frac{f_M}{r_M}$, in Fig. 3. This ratio represents the marginal rate of return to the AFS facility investment. We observe that the return on the investment for the myopic planner is always lower than that of the optimal planner. The difference between these two ratios increases with σ for a high γ (as shown in Fig. 3(a)) but decreases for a low γ (as shown in Fig. 3(c)). This illustrates that the over-commitment in facility deployment adversely affects the myopic planner when γ and σ are both either high or low.



(a) Fast-learning market(γ_H)

(b) Moderate market (γ_M)

(c) Slow-learning market (γ_L)

Fig. 3 Over-commitment in AFS investment with lack of foresight

6 Conclusions

In this paper, we study the two-stage alternative fuel service location problem in the presence of endogenous demand learning time for the alternative fuel charging service. In particular, a service provider sequentially deploys charging stations over the two stages when the service demand rate is learned at the end of the first stage. The length of the first stage (demand learning time) is determined by the service provider's deployment action in this stage. We model the problem as a two-stage nonlinear integer program and propose an efficient and effective solution method. The proposed algorithm provides a framework to achieve a desired error rate of accuracy in the optimal solution.

Using the model, we first study the trade-off between active learning and deferred commitment in AFS deployment strategy under different market characteristics. For the fast-learning markets, we recommend deferred commitment as uncertainty in the service demand rate increases. In contrast, for the slow-learning markets, active learning is recommended as uncertainty in the service demand rate increases. Next, we show that the value of having foresight in deployment of AFS becomes crucial when the learning speed and uncertainty are both either high or low. The lack of such foresight typically results in an over-commitment in facility investment while earning a lower expected profit.

Finally, we note that this research can be extended in several ways. First, it would be interesting to consider the site-specific service demand rate θ_i. Although the demand rates in the same market will likely to be correlated, relaxing the uniform demand rate will certainly enrich the proposed model. Another possible research direction is to study a multi-stage version of the problem where the service demand rate (or its distribution) is partially learned over time. One may consider a Bayesian learning scheme in implementing this research. We leave these promising research directions as future work.

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